Stochastic Modeling of the Decay Dynamics of Online Social Networks

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ABSTRACT

The dynamics of online social networks (OSNs) is not limited to a growth process, instead, a decay process sometimes occurs. In the last decade, many online social networks, like **MySpace** and **Orkut**, suffered from decaying process and they ended with being out of service. Thus, understanding this decay process is crucial for many aspects that include: (1) Engineering a resilient network, (2) Accelerating the dissolving of some networks, and (3) Predicting users leave/inactivity dynamics. In this work we are interested in modeling and understanding the decay dynamics in OSNs in order to able to handle the aforementioned three aspects. Here, we present a probabilistic model that captures the dynamics of the social decay due to the inactivity of the members in a social network. The model is proved to be ***submodular***, which implies achieving the model optimization in a reasonable performance. We provided preliminary results and investigated some properties of real networks under decay process and compare it to our model's results. The results showed, at the macro level of the networks, that there is a match between the properties of the real networks under decay dynamics and our model's results.

MOTIVATION

The following networks in Fig. 1 are the networks of the "Business Startup" websites of the stack exchange. The networks show a decay before the website was closed due to lack of activity.

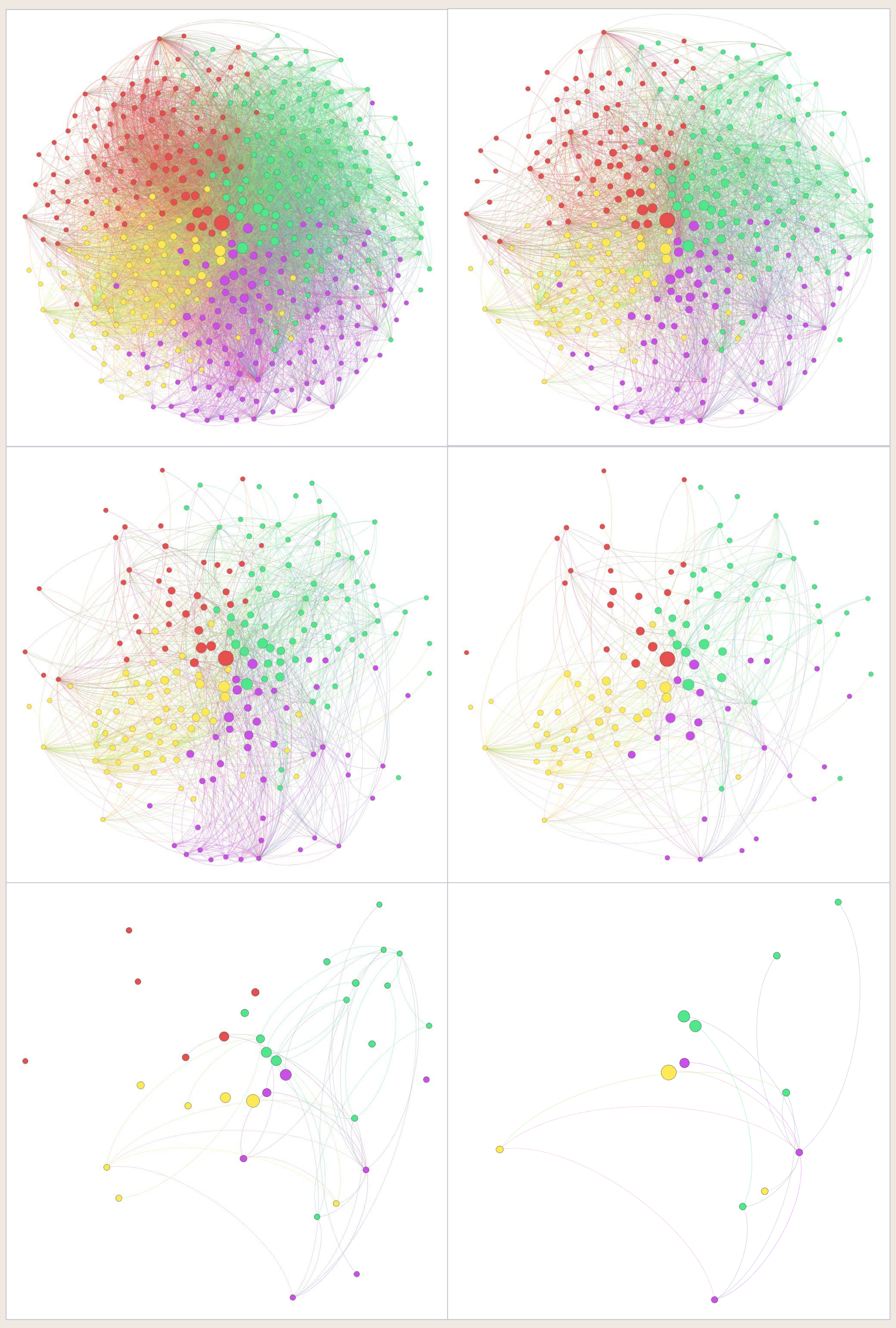

Figure 1

The networks in the Fig. 1 are a representation of the real data of the "Business Startups" website social network between Oct-2009 and Nov-2011. The figure also shows that a network representation of the interaction is a good abstraction (model) choice to capture the decay of the interaction of among the members of the website.

THE MODEL

The following figure shows an illustration of the model. The color of the nodes represents how likely a node will leave in the future, where white nodes are very unlikely to leave and the level of grayness correlates with the probability to leave. Whenever a node leaves the network, it is marked as black, all its edges are removed, and all of its neighbors get affected by its leave by increasing their leave probability. The dotted edges are the removed edges.

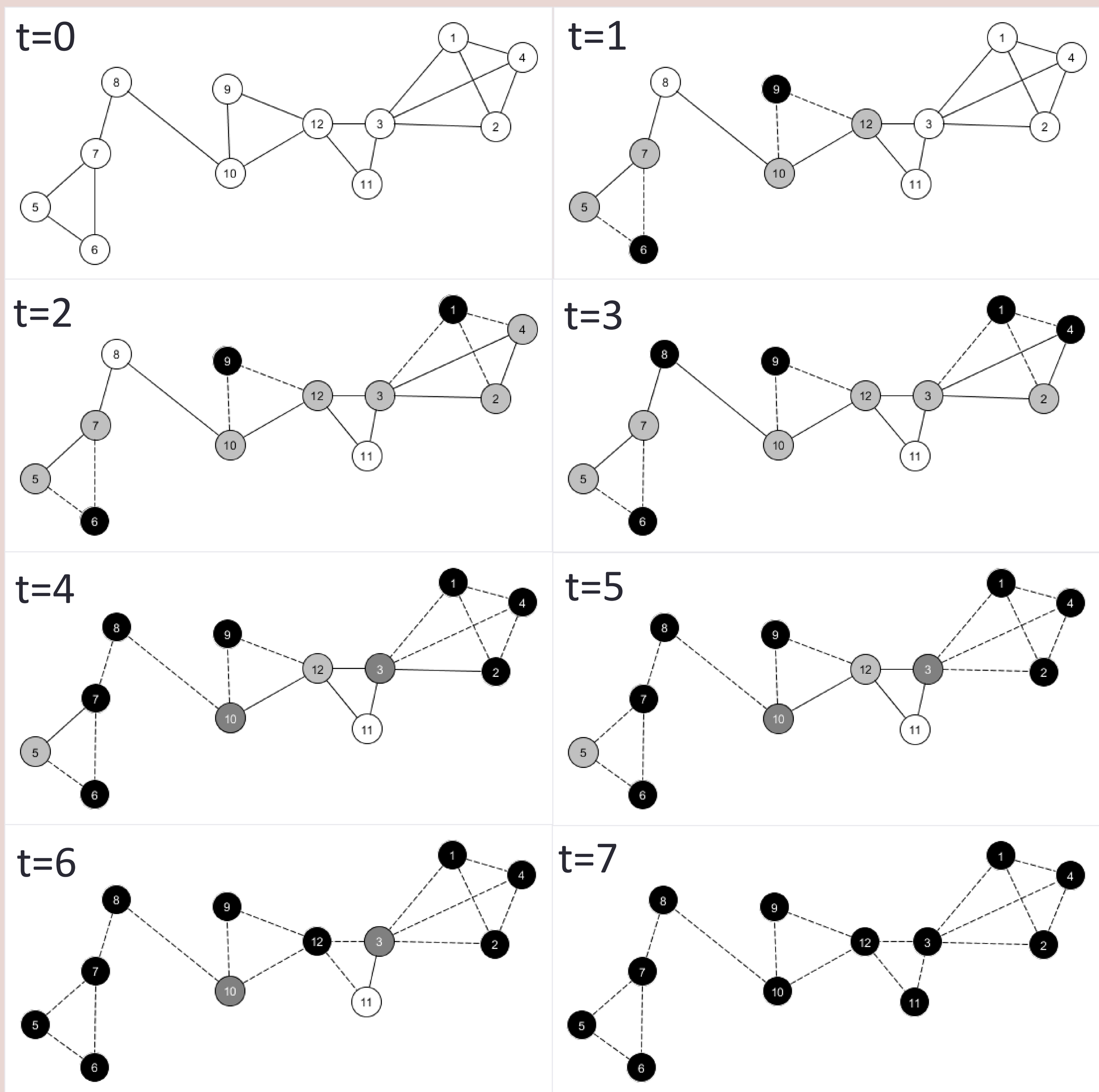

Figure 2

Node Leave effect: Figure 3 shows how a node *v* affects all of its neighbors when it leaves. At *t-2*, the node *v* has a leave probability π_v^{t-2} which was gained by *v*'s initial leave probability π_v^0 and possible probability gains caused earlier by leaving neighbors, i.e., $\pi_v^{t-2} = \pi_v^0 + \sum_{t=1}^{t=t-3} \Delta\pi_v^t$. At time *t-1*, the node *v* leaves the network affecting its neighbors by increasing the leave probability of nodes 1,2,4,5. Here we assume that the tie strength between *v* and the nodes 1,2,5 is greater than the tie strength (δ) between *v* and 4. That is why the nodes 1,2,5 gain more leave probability than node 4, which is represented by a darker color of nodes 1,2,5. Equation 1 captures this dynamics in the leave probabilities incorporating the tie strength between the nodes.

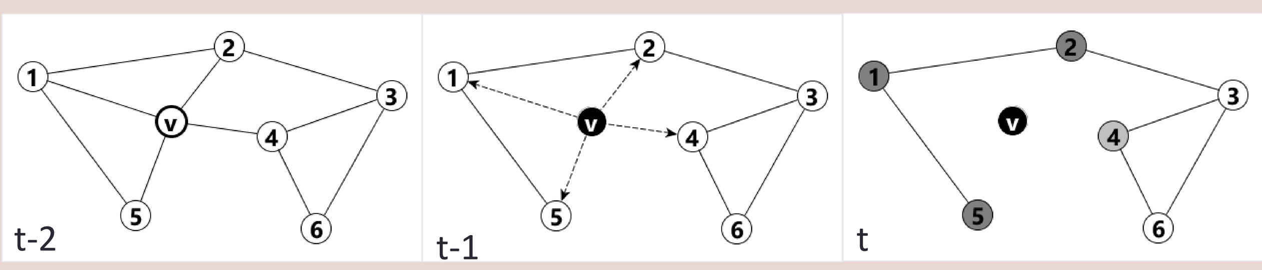

Figure 3

$$\Delta\pi^t(v) = \sum_{w \in left} 1 - (1 - \pi_v^{t-1})(1 - \delta_{v,w}^{t-1}) \qquad (1)$$

Neighbors leave effect: Figure 4 shows how a node *w* is affected by the leave of its neighbors. At *t-2*, the nodes 1,4 have leave probabilities π_1^{t-2} and π_4^{t-2}, respectively, which were gained by the nodes' initial leave probabilities π_1^0 and π_4^0 and possible earlier probability gains. At time t-1, the nodes 1,4 leave the network affecting their neighbors, including node *w*. The leave of nodes 1,4 increases the leave probability of the node *w* at t. Equation 2 captures this dynamics in the leave probabilities.

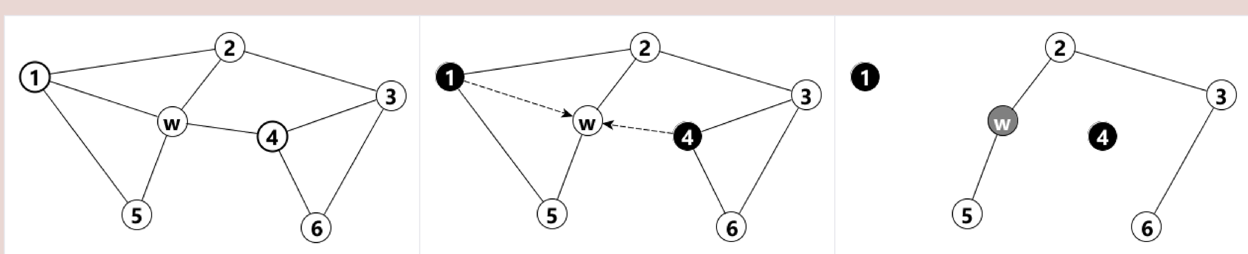

Figure 4

$$\Delta\pi_w^t = 1 - [\underbrace{(1 - \xi_w^{t-1})}_{\text{Assures leave}}(\underbrace{\prod_{u \in \overline{\Gamma}_w^{t-1}} (1 - \pi_u^{t-1})}_{\text{Leave probabilities effect}})(\underbrace{\prod_{u \in \overline{\Gamma}_w^{t-1}} (1 - \delta_{u,w}^{t-1})}_{\text{Tie strength effect}})]$$

$$= 1 - [(1 - \xi_w^{t-1})(\prod_{u \in \overline{\Gamma}_w^{t-1}} (1 - \pi_u^{t-1})(1 - \delta_{u,w}^{t-1}))] \qquad (2)$$

RESULTS

Theorem: Model's equations 1 and 2 are *monotone* and *submodular*.

Implications: The optimization of the model is viable.
The maximization problem under the settings of the model is defined as: Select a set of nodes **A** of maximum size *k* such that the number of left nodes at time t+1 is maximum.

$$Maximize \sum_{v \in V(G)_t} \Delta\pi^t(v) = \sum_{v \in V(G)_t} \sum_{w \in \Gamma_v^t} 1 - (1 - \pi_v^{t-1})(1 - \delta_{v,w}^{t-1})$$
$$Subject\ to\ |\mathcal{A}| \le k, \mathcal{A} \subseteq V(G^t)$$

Conversely, the minimization is defined as: Select a set of nodes **A** of maximum size *k* such that the number of left nodes at time t + 1 is minimum.

$$Minimize \sum_{v \in V(G)_t} \sum_{w \in \Gamma_v^t} 1 - (1 - \pi_v^{t-1})(1 - \delta_{v,w}^{t-1})$$
$$Subject\ to |\mathcal{A}| \le k, \mathcal{A} \subseteq V(G)$$

The analysis result of real decayed stack exchange sites compared to alive ones. Figures 5 and 6 show the difference of the interaction distribution between users for the comments and the upvotes, respectively. Bold markers are for decayed (closed) stack exchange websites. The nonbold markers are for alive sites.

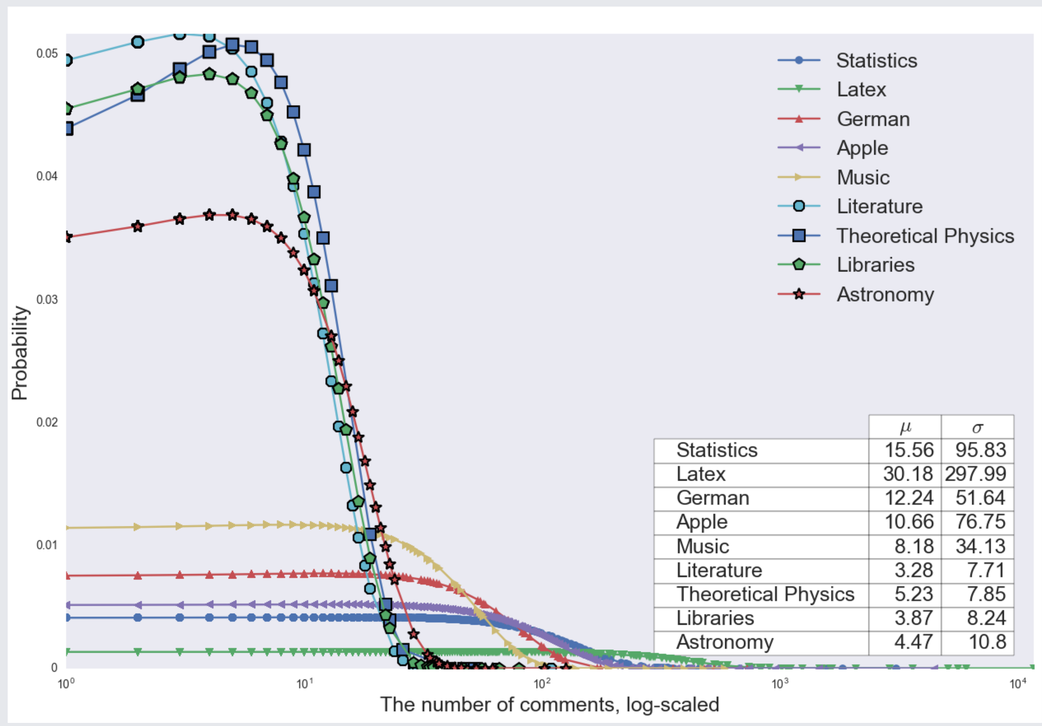

Figure 5

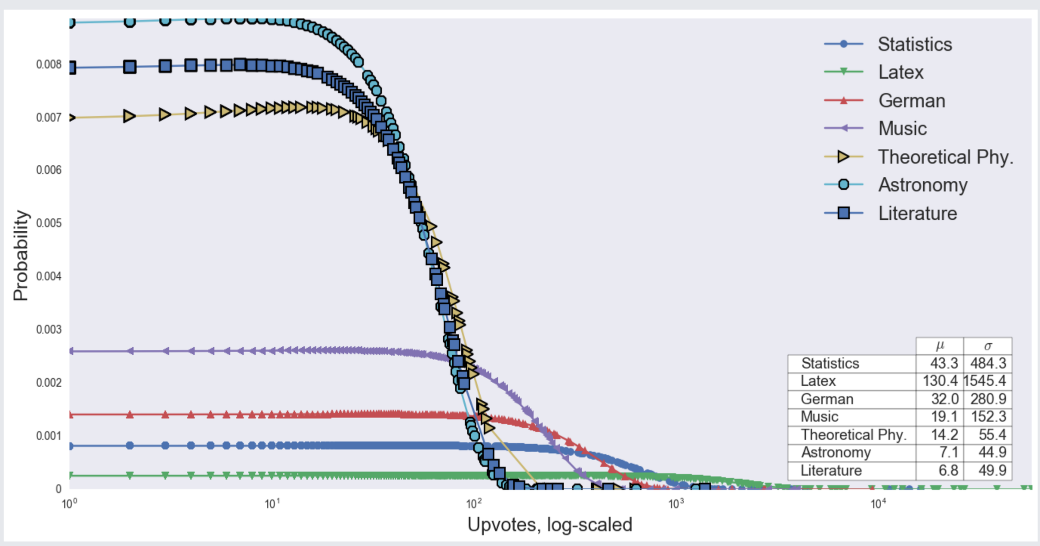

Figure 6

APPLICATION AND FUTURE WORK

The model can be utilized in the following scenarios:

- detecting leave cascade
- Maximizing the leave effect for malicious networks
- Engineering resilient networks

The next step in our work is to design algorithms to achieve and test the optimization properties of the model.

REFERENCES

- Abufouda, Mohammed, and Katharina A. Zweig. "**Stochastic Modeling of the Decay Dynamics of Online Social Networks.**" Complex Networks VIII (2017)
- Abufouda, Mohammed, and Katharina A. Zweig. "**A Theoretical Model for Understanding the Dynamics of Online Social Networks Decay.**" *arXiv preprint arXiv:1610.01538* (2016).